Vox Populi Midterm 1 Review (Spring 2018)

Math 1220

Sketching Level Curves. To sketch level curves of $f(x, y) = c$ for some constant c, sketch the locus of all points in the xy-plane that satisfy the equation.

Challenge 1. Sketch the level curves of the function $f(x, y) = 4x + 6y$ corresponding to $z = -2, -1, 0, 1, 2$.

Volumes and Areas. If it is not given, half the battle is finding the right region to integrate! Recall the "general method" for finding where a region bounded by curves is. If a region is bounded by curves $y = f(x)$ and $y = g(x)$, solve $f(x) = g(x)$ and figure out which function is bigger on each side of the zeros.

Challenge 2. Find the volume of the solid of revolution obtained by revolving the region bounded by the curves $y = e^x$ and $y = x^2 + \frac{1}{2}$ from $x = 0$ to $x = 1$.

Using Trig Identities. The study guide says, "Because the ordered pair $(\cos\theta, \sin\theta)$ lies on the unit circle for which $x^2 + y^2 = 1$ is an equation, we obtain *the fundamental trig identity*

$$\cos^2\theta + \sin^2\theta = 1$$

You are expected to know the preceding identity as well as the following quick corollaries:

$$1 + \tan^2\theta = \sec^2\theta \text{ and } \cot^2\theta + 1 = \csc^2\theta$$

where the former is obtained from the fundamental trig identity by dividing both sides by $\cos^2$ and the latter is obtained by dividing both sides by $\sin^2\theta$." Why does it say this? Consider the following question from practice exam S16.

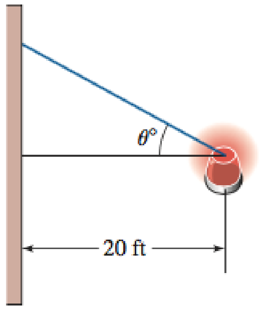
Challenge 3. Compute the integral $\int \cos(x)(1 - \cos^2(x))^{10}\, dx$.

Or this question from practice exam F16

Challenge 4. Prove the identity $\tan^2\theta - \sin^2\theta = \sin^2\theta\tan^2\theta$ by starting with the expression $\tan^2 - \sin^2\theta$ on the left and showing how it can be transformed, using definitions and known identities, into the expression on the right.

Trig Related Rates. The study guide says "know how to solve related rates problems."

Challenge 5. (Textbook 12.3 exercise 63). A police cruiser hunting for a suspect pulls over and stops at a point 20 ft from a straight wall. The flasher on top of the cruiser revolves at a constant rate of 90°/sec, and the light beam casts a spot of light as it strikes the wall. How fast is the spot of light moving along the wall at a point 30 ft from the point on the wall closest to the cruiser.



Solving problems with given info. You can get questions where you are given some information you need and then asked to solve a question.

Challenge 6. (F17 8). Suppose that f is twice differentiable with continuous second derivative. The following table gives some values of f and f'.

x	-1	0	1	2	e	3	5	e^2
$f(x)$	3	-1	7	1	5	e^2	5	0
$f'(x)$	1	-3	4	2	0	-5	11	3

Use the table above to compute the following. If there is not enough information to solve the problem, write "Not Enough Info".

(a) $\displaystyle\int_{-1}^{2} \frac{f'(x)}{f(x)^2}\,dx$

(b) $\displaystyle\int_{0}^{5} x f''(x)\,dx$

Unfamiliar word problems. Do not worry! You know enough to solve unfamiliar problems.

Challenge 7. (F17 11). A pulse traveling along a string can be modeled by the equation

$$u(x,t) = \sin(x - vt)$$

where u represents the amplitude of the wave, x is the position of the string, t is the time, and v is the speed of the pulse.

(a) Find $\dfrac{\partial^2 u}{\partial t^2}$ and $\dfrac{\partial^2 u}{\partial x^2}$

(b) We say that u satisfies the *wave equation* if $\dfrac{\partial^2 u}{\partial t^2} = v^2 \dfrac{\partial^2 u}{\partial x^2}$. Using (a), does $u(x,t)$ satisfy the wave equation?

(c) (Extra; my own.) Sketch level curves for $u(x,t) = -1, 0, 1$ when $v = 1$.

Remember that partial derivatives can also be mixed!

Challenge 8. (S16 2). Consider the function $f(x,y) = \sin(y^3 - x) + e^{xy}$. Find $\dfrac{\partial^2 f}{\partial x \partial y}$.

Finally, you can be asked to combine your knowledge in new ways.

Challenge 9. (S16 7). Let $f(x) = \dfrac{1}{2}\sin^2(x)$ and $g(x) = \dfrac{1}{2}\cos(x)$. Find the x-coordinates of all the points where the graphs of f and g have parallel tangent lines. Hint: parallel lines have the same slope.