Name:________________________

Maximization, Minimization, and OLS Homework

Math 1220

Challenge 1. C&G Imports imports two brands of white wine, one from Germany and the other from Italy. The German wine costs \$4/bottle, and the Italian wine costs \$3/bottle. It has been estimated that if the German wine is sold at p dollars/bottle and the Italian wine is sold for q dollars/bottle, then

$$2000 - 150p + 100q \text{ bottles of German wine and}$$

$$100 + 80p - 120q \text{ bottles of Italian wine}$$

will be sold each week. Determine the unit price for each brand that will allow C&G to realize the largest possible weekly profit.

Challenge 2. Remember sigma notation, which is an abbreviation for a sum,

$$\sum_{i=0}^{n} i = 0 + 1 + 2 + \cdots + n$$

Compute the following sums

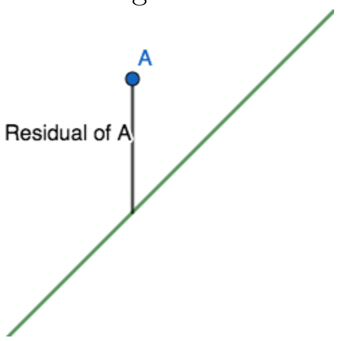
(a) $\displaystyle\sum_{i=1}^{7} i$

(b) $\displaystyle\sum_{i=2}^{8} i$

(c) $\displaystyle\sum_{i=2}^{5} (i-1)^2$

Challenge 3. Let us show the normal equations for Ordinary Least Squares actually give us a "best-fit" line minimizing the squares of the "residuals" between n points, say $(x_1, y_1), \ldots, (x_n, y_n)$ and the line.

(a) What is the length of a "residual" between a point (x_0, y_0) and a line $y = mx + b$?



(b) Let $y = f(x) = mx + b$ be our line and let $d(m, b)$ be the sum of the squares of the n residuals between our line and our data points. Write a formula for $d(m, b)$ in sigma notation.

(c) In order to find our line of best fit, we must minimize $d(m, b)$. Compute the first-order partial derivatives of d and set them equal to zero.

(d) Now, we need to show that our critical point (m, b) is a relative minimum. Compute the second partial derivatives d_{mm}, d_{mb}, and d_{bb} and apply the second derivative test. To show $D(m, b) > 0$, you can use the famous *Cauchy inequality*

$$\left(\sum_{i=1}^{n} a_i^2\right)\left(\sum_{i=1}^{n} b_i^2\right) \geq \left(\sum_{i=1}^{n} a_i b_i\right)^2 \quad \text{for all } a_i \geq 0, b_i \geq 0$$