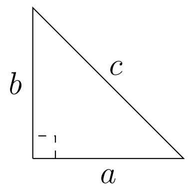
Related Rates Worksheet

Math 1210

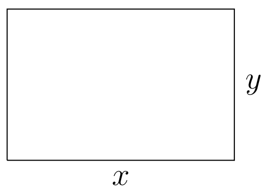
Some Useful Geometric Facts for Common Related Rates Problems

Pythagorean Theorem



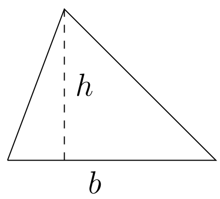
Pythagorean Theorem: $a^2 + b^2 = c^2$

Area, Perimeter, Circumference Formulas

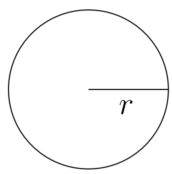


$Area = xy$

$Perimeter = 2x + 2y$



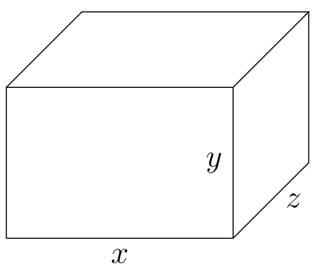
$Area = \frac{1}{2}bh$



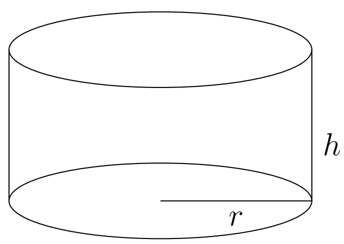
$Area = \pi r^2$

$Circumference = 2\pi r$

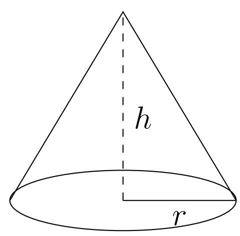
Volume Formulas



$Volume = xyz$



$Volume = \pi r^2 h$



$Volume = \frac{1}{3}\pi r^2 h$

General Guidelines for Solving Related Rates Problems

(a) Assign a variable to each quantity. Draw a diagram if applicable.

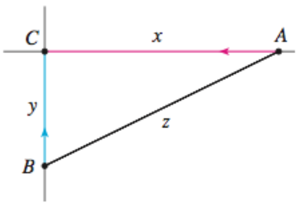
(b) Write down what we are trying to find as well as the given values of the variables and the given rates of change.

(c) Write down any equation(s) that relate the variables.

(d) Implicitly differentiate equation(s).

(e) Solve for desired rate of change. Then state the final answer and remember to include units.

Challenge 1. Car A is traveling west at 50 mi/h and car B is traveling north at 60 mi/h. Both are headed for the intersection of the two roads (denoted C). At what rate are the cars approaching each other when car A is 0.3 mi and car B is 0.4 mi from the intersection?

(a) 

(b) $\dfrac{dx}{dt} =$ What are we trying to find?

$\dfrac{dy}{dt} =$

(c) What is the relationship between x, y, and z?

(d) Implicitly differentiate the relationship you found above.

(e) Solve the equation above for $\dfrac{dz}{dt}$. Your answer will be in terms of $x, y, z, \dfrac{dx}{dt}$, and $\dfrac{dy}{dt}$. Finally, plug in the desired values $x = 0.3$ mi, $y = 0.4$ mi, the value of z (from the relationship in part (c)), and what you found for $\dfrac{dx}{dt}$ and $\dfrac{dy}{dt}$ in part (a).

Challenge 2. Now, do WebAssign 3.6 some of questions 7–12 for practice.

Challenge 3. Write out your solution on a separate piece of paper using the structure of the solution above as a template. Suppose that water is flowing out of a hole at the bottom of a cone into a cylinder located below the bottom of the cone. Suppose that the height of the water in the cone is decreasing at a constant rate of 2 cm/sec. The cone has radius 10 cm and height 20 cm. The cylinder has radius 30 cm and height 70 cm. For simplicity, assume that as soon as water leaves the hole of the cone it instantaneously goes into the cylinder. In other words, it takes 0 time for the water to fall from the hole into the cylinder. What is the rate of change of the height of the water in the cylinder when the height of the water in the cone is 4 cm? **(Hint: Use Similar Triangles)**

3 (a)