Name:________________________

Worksheet. Lots of Derivatives!

Math 1210

Challenge 1. Air is being pumped into a spherical weather balloon. At any time t, the volume of the balloon is $V(t)$ and its radius is $r(t)$. Recall that, for a sphere, $V = \frac{4}{3}\pi r^3$.

(a) What do the derivatives $\frac{dV}{dr}$ and $\frac{dV}{dt}$ represent?

(b) Express $\frac{dV}{dt}$ in terms of $\frac{dr}{dt}$.

Challenge 2. Write $|x| = \sqrt{x^2}$ and use the chain rule to show

$$\frac{d}{dx}|x| = \frac{x}{|x|}$$

Also recall how to get the answer using the limit definition of the derivative.

Challenge 3. The cost function for production of a commodity is

$$C(x) = 339 + 25x - 0.09x^2 + 0.0004x^3$$

(a) Find and interpret $C'(100)$

(b) Compare $C'(100)$ with the cost of producing the 101st item, that is $C(101) - C(100)$.

Challenge 4. Consider a tank holding 5000 gallons of water which drains from the bottom and is completely empty after 40 minutes of draining. Torricelli's Law gives the volume V of water remaining in the tank after t minutes as

$$V(t) = 5000\left(1 - \frac{t}{40}\right)^2, \quad 0 \leq t \leq 40$$

Find the rate at which water is draining from the tank after (a) 5 minutes, (b) 10 minutes, (c) 20 minutes, and (d) 40 minutes. At what time is the water flowing out the fastest? The slowest?

Challenge 5. A company makes computer chips from square wafers of silicon. It wants to keep the side length of a wafer very close to 15mm and it wants to know how the area $A(x)$ of a wafer changes when the side length x changes. Find $A'(15)$ and explain its meaning.