

MATH 1210, SECTION 011
COMMON MISTAKES WITH LIMITS

Compute the following limits or write DNE if the limit does not exist.

(a) $\lim_{x \rightarrow -7} \frac{x^2 + 12x + 35}{x + 7}$

(b) $\lim_{x \rightarrow 0} \frac{x^2 + x}{x + 10}$

Here is how to **CORRECTLY** write the answer to these questions:

(a) **Solution:**

$$\begin{aligned}\lim_{x \rightarrow -7} \frac{x^2 + 12x + 35}{x + 7} &= \lim_{x \rightarrow -7} \frac{(x + 5)(x + 7)}{x + 7} \\ &= \lim_{x \rightarrow -7} x + 5 \\ &= -7 + 5 \\ &= -2\end{aligned}$$

(b) **Solution:**

$$\lim_{x \rightarrow 0} \frac{x^2 + x}{x + 10} = \frac{0^2 + 0}{0 + 10} = \frac{0}{10} = 0$$

Here is a list of common ways to **INCORRECTLY** answer these question:

- (1) **Scribbling scratch work on the side and writing final answer without showing any logical steps.**
- (2) **Failure to use limit signs.**

Question (a) example solution:

$$\begin{aligned}\lim_{x \rightarrow -7} \frac{x^2 + 12x + 35}{x + 7} &= \frac{(x + 5)(x + 7)}{x + 7} \\ &= x + 5 \\ &= -2\end{aligned}$$

- (3) **Failure to write the expression after the limit sign.**

Question (a) example solution:

$$\begin{aligned}\lim_{x \rightarrow -7} \frac{x^2 + 12x + 35}{x + 7} &= \lim_{x \rightarrow -7} \frac{(x + 5)(x + 7)}{x + 7} \\ &= \lim_{x \rightarrow -7} x + 5 \\ &\lim_{x \rightarrow -7} = -2\end{aligned}$$

The last equality is nonsensical. The limit sign with no expression next to it is meaningless.

(4) **Failure to use equalities.**

Question (a) example solution:

$$\begin{aligned}\lim_{x \rightarrow -7} \frac{x^2 + 12x + 35}{x + 7} & \lim_{x \rightarrow -7} \frac{(x + 5)(x + 7)}{x + 7} \\ & \lim_{x \rightarrow -7} x + 5 \\ & -7 + 5 \\ & \boxed{-2}\end{aligned}$$

(5) **Using arrows instead of equalities.**

Question (a) example solution:

$$\begin{aligned}\lim_{x \rightarrow -7} \frac{x^2 + 12x + 35}{x + 7} & \implies \lim_{x \rightarrow -7} \frac{(x + 5)(x + 7)}{x + 7} \\ & \implies \lim_{x \rightarrow -7} x + 5 \\ & \implies -7 + 5 \\ & \implies -2\end{aligned}$$

In mathematics, if P and Q are statements, then $P \implies Q$ means P implies Q . It does not make sense to say

$$\lim_{x \rightarrow -7} \frac{x^2 + 12x + 35}{x + 7} \implies \lim_{x \rightarrow -7} \frac{(x + 5)(x + 7)}{x + 7}$$

because the things on the left and right of the arrows are not statements, they are numbers. An example of an appropriate time to use an arrow is the following:

$$x + 1 = 6 \implies x = 5$$

This time the arrow makes sense because $x + 1 = 6$ and $x = 5$ are both statements. $x + 1 = 6$ is the statement “ x plus one is equal to 6” and $x = 5$ is the statement “ x is equal to 5”.

(6) **Misplaced equal signs.**

Question (a) example solution:

$$\begin{aligned}\lim_{x \rightarrow -7} & = \frac{x^2 + 12x + 35}{x + 7} \quad \lim_{x \rightarrow -7} = \frac{(x + 5)(x + 7)}{x + 7} \\ & \lim_{x \rightarrow -7} = x + 5 \\ & = -7 + 5 \\ & = -2\end{aligned}$$

The equal sign should always go before the limit sign, not between the limit sign and the expression.

(7) **Setting limit equal to $\frac{0}{0}$.**

Question (a) example solution:

$$\lim_{x \rightarrow -7} \frac{x^2 + 12x + 35}{x + 7} = \frac{(-7)^2 + 12(-7) + 35}{-7 + 7} = \frac{0}{0}$$

The limit of a function either does not exist or is a number. It does not make sense to say that the limit of a function is equal to $\frac{0}{0}$. Remember if you plug in and get $\frac{0}{0}$ it means that more work is required. You must perform algebraic operations to reduce the expression and compute the limit or determine it does not exist.

(8) **Getting $\frac{0}{a}$ where a is some non-zero real number and determining that the limit DNE.**

Question (b) example solution:

$$\lim_{x \rightarrow 0} \frac{x^2 + x}{x + 10} = \frac{0^2 + 0}{0 + 10} = \frac{0}{10} = DNE$$

If a is a non-zero real number, then $\frac{0}{a} = 0$.