

**MATH 1210, SECTION 002**  
**SUMMARY OF DERIVATIVE APPLICATION PROCEDURES**  
**10/16/2017**

**Procedure for determining intervals of increase/decrease of “nice” functions.**

- (1) Compute  $f'$ . Then find the  $x$  values where  $f'(x) = 0$  or  $f'$  is discontinuous. Plot these numbers on the number line.
- (2) In each open interval between the numbers plotted in part (1), pick a number  $c$  and plug it into  $f'$ .
  - (a) If  $f'(c) > 0$ , then  $f$  is increasing on the corresponding open interval containing  $c$ . Write a plus sign above this interval.
  - (b) If  $f'(c) < 0$ , then  $f$  is decreasing on the corresponding open interval containing  $c$ . Write a minus sign above this interval.
- (3) Write down the intervals, in interval notation, on which  $f$  is increasing. If there is more than one interval write “and” between rather than  $\cup$ . Do the same thing for the intervals on which  $f$  is decreasing.

**First Derivative Test: Procedure for determining relative max(s) and relative min(s) of “nice” functions.**

- (1) Determine the critical numbers of  $f$ . In other words, calculate  $f'(x)$ . Then, find the values  $x$  **IN THE DOMAIN OF**  $f$  for which  $f'(x) = 0$  or  $f'(x)$  DNE. Plot these numbers on the number line. Notice the difference in this step and step (1) of the previous procedure.
- (2) In each open interval between the numbers plotted in part (1), pick a number  $c$  and plug it into  $f'$ . If  $f'(c) > 0$ , put a plus sign above the corresponding interval. If  $f'(c) < 0$  put a minus sign above the corresponding interval. If  $x$  is a critical number and the sign chart zoomed in at  $x$  looks like:

(a) 
$$\begin{array}{c} + \quad - \\ | \\ \hline x \end{array}$$
 Then  $f$  has a relative max of  $f(x)$  at  $x$ .

(b) 
$$\begin{array}{c} - \quad + \\ | \\ \hline x \end{array}$$
 Then  $f$  has a relative min of  $f(x)$  at  $x$ .

(c) 
$$\begin{array}{c} + \quad + \\ | \\ \hline x \end{array} \quad \text{or} \quad \begin{array}{c} - \quad - \\ | \\ \hline x \end{array}$$

Then  $f$  has a neither a relative max nor a relative min at  $x$ .

- (3) Write down the relative max(s) and relative min(s).

**Second Derivative Test: An alternate (less powerful) procedure for determining relative max(s) and min(s) of “nice” functions.**

- (1) Compute  $f'$  and  $f''$ . Find all  $x$  values where  $f'(x) = 0$ .
- (2) For each value  $x$  from part (1), plug  $x$  into  $f''$ .
  - (a) If  $f''(x) < 0$ , then  $f$  has a relative max of  $f(x)$  at  $x$ .
  - (b) If  $f''(x) > 0$ , then  $f$  has a relative min of  $f(x)$  at  $x$ .
  - (c) If  $f''(x) = 0$  or  $f''(x)$  DNE, then we cannot conclude anything. Therefore, this test is less powerful. It will not always tell us as much as the first derivative test.
- (3) Write down the relative max(s) and relative min(s).

**Procedure for determining intervals of concavity of “nice” functions.**

- (1) Compute  $f''$ . Then find the values of  $x$  where  $f''(x) = 0$  or  $f''(x)$  DNE. Plot these numbers on the number line.
- (2) In each open interval between the numbers plotted in part (1), pick a number  $c$  and plug it into  $f''$ .
  - (a) If  $f''(c) > 0$ , then the graph of  $f$  is concave upward on the corresponding interval containing  $c$ . Write a plus sign above this interval.
  - (b) If  $f''(c) < 0$ , then the graph of  $f$  is concave downward on the corresponding interval containing  $c$ . Write a minus sign above this interval.
- (3) Write down the intervals, in interval notation, on which  $f$  is concave upward. If there is more than one interval write “and” between rather than  $\cup$ . Do the same thing for intervals on which  $f$  is concave down.

**Procedure for determining inflection points of “nice” functions.**

- (1) Compute  $f''$ . Find the values  $x$  **IN THE DOMAIN OF**  $f$  for which  $f''(x) = 0$  or  $f''(x)$  DNE.
- (2) In each open interval between the numbers plotted in part (1), pick a number  $c$  and plug it into  $f''$ . If  $f''(c) > 0$ , put a plus sign above the corresponding interval. If  $f''(c) < 0$  put a minus sign above the corresponding interval. If  $x$  is a number from part (1), and the sign chart zoomed in at  $x$  looks like:

(a) 
$$\begin{array}{c} + \quad - \\ | \\ \hline x \end{array} \quad \text{or} \quad \begin{array}{c} - \quad + \\ | \\ \hline x \end{array}$$

Then  $(x, f(x))$  is an inflection point.

(b) 
$$\begin{array}{c} + \quad + \\ | \\ \hline x \end{array} \quad \text{or} \quad \begin{array}{c} - \quad - \\ | \\ \hline x \end{array}$$

Then  $(x, f(x))$  is not an inflection point.

- (3) Write down the inflection point(s).