Name:________________________

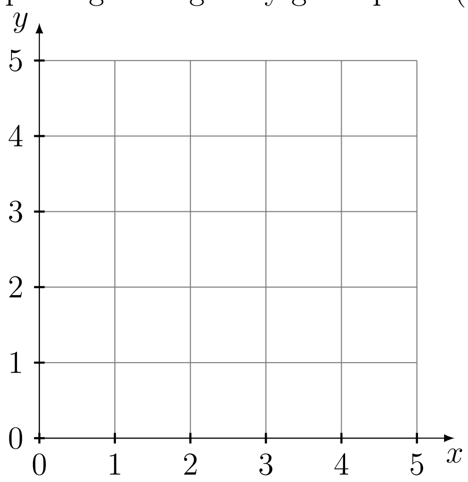
Notesheet. Section 9.4: Euler's Method

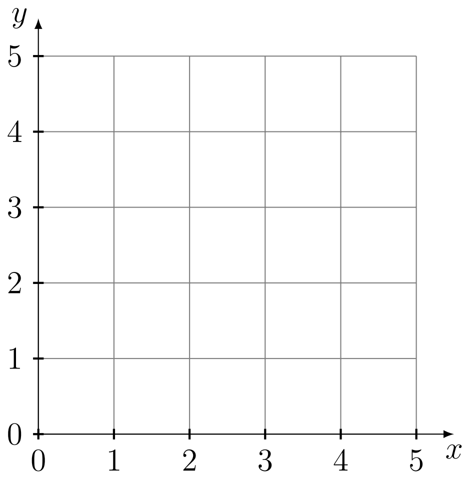
Math 1220

We cannot solve every 1st order differential equation of the form

$$y' = F(x, y)$$

However, given a specific point, we know the slope of the tangent line of the solution $y(x)$ passing through any given point (x_0, y_0).





Using this information, we can approximate solutions to the IVP

$$y' = F(x, y), y(x_0) = y_0$$

Challenge 1. Approximate the value of $y(4)$ where $y(x)$ is the solution to the IVP

$$y' = x - y, y(0) = 1$$

Theorem 2 (Euler's Method)**.** Given a first order IVP of the form

$$y' = F(x, y),\ y(x_0) = y_0$$

we can approximate $y(b)$ in n steps by

(i) Finding the step size $h = \dfrac{b - x_0}{n}$

(ii) Determining the x-values $x_i = x_{i-1} + h$ for $1 \leq i \leq n$.

(iii) Determining the y-values

$$\begin{cases} y_1 = \\ y_2 = \\ \vdots \\ y_i = \end{cases}$$

Challenge 3. Use Euler's method with $n = 1$, $n = 2$, and $n = 4$ to obtain an approximation of the solution of the IVP

$$y' = xy, y(1) = 2$$

when $x = 5$. Which approximation is the most accurate?