

Notesheet. Section 9.2: Separation of Variables

Math 1220

Definition 1. A separable differential equation is a differential equation that can be written in the form

$$\frac{dy}{dx} =$$

Theorem 2. To solve a separable differential equation, we solve

$$\int \frac{dy}{g(y)} = \int f(x) dx$$

Challenge 3. Solve the following differential equations using the method of separation of variables

(a) $y' = xy^2$

(b) $y' = \frac{6x^2}{2y + \cos(y)}$

(c) $y' = \frac{y}{x}$

(d) $y' = \frac{xy}{x^2 + 1}, y(0) = 1$

Challenge 4. A radio active isotope has initial mass 100g. It decays at a rate proportional to its current mass and 5 years later, its mass is 60g.

(a) Find an expression giving $Q(t)$, that is, the amount of the isotope (in grams) after t years.

(b) Find the half-life of this isotope, ie, find T such that $Q(T) = \frac{1}{2}Q(0)$.