Name:_____

Notesheet. Section 9.1: Differential Equations

Math 1220

Definition 1.

- A differential equation is an equation involving

- A solution of a differential equation is

- A general solution of a differential equations is

- A particular solution is

Challenge 2. Consider the differential equation $y+y'' = 0$ (from the "trig identity homework").

(a) What are 3 solutions to this differential equation?

(b) What is the general solution to this differential equation?

Definition 3. An initial value problem (IVP) is problem that requires solving a differential equation with some "initial conditions."

Challenge 4. (a) Given that $y = Ce^{-x} + x - 1$ is a general solution of $y' + y = x$, find the particular solution of the IVP

$$y' + y = x \text{ and } y(0) = 2$$

(b) Given that $y = C_1x^3 + C_2x^2$ is a general solution of $x^2y'' - 4xy' + 6y = 0$, find the particular solution of the IVP

$$x^2y'' - 4xy' + 6y = 0, y(2) = 0, y'(2) = 4$$

Theorem 5. Let $Q(t)$ be the amount of some quantity at time t. Recall tha the rate of change of Q is given by $\dfrac{dQ}{dt}$. Then,

(a) Q increases (decreases) at a fixed rate $k \iff \dfrac{dQ}{dt} =$

(b) Q grows (decays) at a rate proportional to some quantity $A \iff \dfrac{dQ}{dt} =$

(c) Q grows (decays) at a rate jointly proportional to A and $B \iff \dfrac{dQ}{dt} =$

Challenge 6. Write out (but do not solve) an initial value problem for each of the following situations.

(a) The world population at the beginning of 2018 was 7.6 billion. Setup a model for the world population over time assuming that the population will continue to grow at a rate of approximately 2% per year.

(b) A radioactive substance decays at a rate directly proportional to the current mass of the substance. Setup a model for the mass of the substace over time assuming that you start with 10 grams.

(c) During a flu epidemic, 5% of 300 Math 1220 students have contracted influenza at time $t = 0$. The rate at which they contract influenza is jointly proportional to the number of students who already have influenze and the number of students who have yet to be infected. 20% of students have contracted the flu by the 10th day.