Name:________________________

Notesheet. Section 8.7+8.8: Double Integrals + Geometric Applications

Math 1220

Definition 1. The <u>double integral</u> of $f(x, y)$ over a region R is denoted by

$$\iint_R f(x, y)\ dA =$$

and, if $f(x, y) \geq 0$ over R, then

$$\iint_R f(x, y)\ dA =$$

Remark 2. *(a) In practice, double integrals are evaluated using suitable iterated integrals.*

(b) A consequence of our definition is that, for a region R, $\iint_R 1\ dA$ numerically gives

Theorem 3. If R is a rectangular region defined by $a \leq x \leq b$ and $c \leq y \leq d$, then

$$\iint_R f(x, y)\ dA =$$

Challenge 4. Evaluate the double integral $\iint_R 2xy\ dA$ where R is the region defined by the inequalities $0 \leq x \leq 1$ and $0 \leq y \leq 1$.

Theorem 5. (a) Suppose $g_1(x)$ and $g_2(x)$ are continuous functions on $[a, b]$ and the region R is defined by $R = \{(x, y) \mid g_1(x) \leq y \leq g_2(x); a \leq x \leq b\}$. Then,

$$\iint_R f(x, y)\, dA =$$

(b) Suppose $h_1(y)$ and $h_2(y)$ are continuous functions on $[c, d]$ and the region R is defined by $R = \{(x, y) \mid h_1(y) \leq x \leq h_2(y); c \leq y \leq d\}$. Then,

$$\iint_R f(x, y)\, dA =$$

Challenge 6. Let R be the region bounded by $x = -y$ and $x = y$ for $0 \leq y \leq 1$. Evaluate

$$\iint_R 2xy\, dA$$

Challenge 7. Let R be the region bounded by $y = x^2$ and $y = 4$. Evaluate

$$\iint_R y\, dA$$