Notesheet. Section 8.4+8.7: The Method of Least Squares + Double Integrals

Math 1220

Definition 1. Given n data points $(x_1, y_1), \ldots, (x_n, y_n)$, a scatter diagram is

Definition 2. The principle of least squares states that the straight line L that fits n data points best is

The line L obtained in this manner is called the least-squares line, or regression line.

Theorem 3. Given n data points $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$, then the least squares (regression) line for the data is given by the linear equation

$$y = f(x) = mx + b$$

where the constants m and b satisfy the equations

You will see why this is true in homework.

Challenge 4. The following data consists of the quiz grades for five students

Student	Quiz I Grade	Quiz 2 grade
1	$x_1 = 1$	$y_1 = 1$
2	$x_2 = 2$	$y_2 = 3$
3	$x_3 = 3$	$y_3 = 4$
4	$x_4 = 4$	$y_4 = 3$
5	$x_5 = 5$	$y_5 = 6$

Challenge 5. Evaluate the following integrals by integrating with respect to the appropriate variable and treating the other variable as a constant.

(a)
$$\int_0^1 xy^2 dx$$

(b)
$$\int_0^x (x+y) \, dy$$

(c)
$$\int_1^2 \frac{x}{y^2} e^{x/y} \, dy$$

Challenge 6. Now, evaluate the <u>integrals</u> by doing the inside integral and then the outside integral.

(a)
$$\int_0^1 \int_0^1 xy^2 \, dx \, dy$$

(b)
$$\int_0^1 \int_0^x (x+y) \, dy \, dx$$

(c)
$$\int_0^x \int_0^1 (x+y) \, dx \, dy$$