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# Notesheet. Section 8.3 Part 2: Maxima and Minima of Function of Several Variables 

Math 1220

Challenge 1. Consider the function $f(x, y)=\left(1-x^{2}\right) \sin (y)$. Compute the following
(a) $f_{x}\left(0, \frac{\pi}{2}\right), f_{y}\left(0, \frac{\pi}{2}\right)$ and $f_{x}(1,0), f_{y}(1,0)$.
(b) $f_{x x}\left(0, \frac{\pi}{2}\right), f_{x y}\left(0, \frac{\pi}{2}\right)$, and $f_{y y}\left(0, \frac{\pi}{2}\right)$
(c) $f_{x x}(1,0), f_{x y}(1,0)$, and $f_{y y}(1,0)$

Theorem 2 (The Second Derivative Test). Let $f(x, y)$ be a twice differentiable function with continuous second derivatives. Let

$$
D(a, b)=f_{x x}(a, b) f_{y y}(a, b)-\left[f_{x y}(a, b)\right]^{2}
$$

If $f_{x}=0$ and $f_{y}=0$,
(a) $D(a, b)>0$ and $f_{x x}(a, b)<0$, then
(b) $D(a, b)>0$ and $f_{x x}(a, b)>0$, then
(c) $D(a, b)<0$, then
(d) $D(a, b)=0$, then

Challenge 3. Use the second derivative test to determine $f(x, y)=\left(1-x^{2}\right) \sin (y)$ classify the which type of critical point $\left(0, \frac{\pi}{2}\right)$ and $(1,0)$ are.

Challenge 4. Let $f(x, y)=x^{3}-6 x y+y^{3}$. find and classify all the critical points of $f(x, y)$.

Challenge 5. Find the dimensions that minimized the material used to construct an open rectangular box having a fixed volume of 32 cubic inches.


Challenge 6. Find the point $(x, y)$ such that the sum of the distances squared from $(x, y)$ to $(0,0),(2,1),(0,3)$ is minimized. Hint: remember the distance formula from the Pythagorean theorem: $d^{2}=\left(x-x_{0}\right)^{2}+\left(y-y_{0}\right)^{2}$.

