Name:______________________

Notesheet. Section 8.2: Partial Derivatives

Math 1220

Remember that the derivative of $f(x)$ at $x = a$ is the "rate of change of $f(x)$ at $x = a$" and is defined by

$$f'(a) := \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$$

With multivariable functions, "rate of change" is ambiguous if we do not define which input we are changing.

Definition 1. Let $f(x, y)$ be a function in two variables. We define the partial derivatives as

(a) $f_x(x, y) = \dfrac{\partial}{\partial x} f(x, y) =$

(b) $f_y(x, y) = \dfrac{\partial}{\partial y} f(x, y) =$

That is, $f_x(x, y)$ is the rate of change of $f(x, y)$ if x is varied and y is fixed and the opposite for $f_y(x, y)$.

Challenge 2. Let $f(x, y) = x^2 + xy + y^2$. Compute f_x and f_y. Let $g(x, y) = e^{x^2} \sin(y)$. Compute g_x and g_y.

Remark 3. *(a) We compute partial derivatives, say $\frac{\partial}{\partial x} f(x, y)$, by letting the variable we are differentiating vary and fixing all the others, so in this case, letting x vary and pretending y is constant.*

(b) The chain rule, product rule, and the quotient rule all still apply for partial derivatives.

Challenge 4. Evaluate f_x and f_y for the following functions

(a) $f(x, y) = \ln(7 + xy^2)$

(b) $f(x, y) = \dfrac{x - y}{x + y}$

Theorem 5. Just like $f'(a) = 0 \iff$ the tangent line of the graph of $f(x)$ is horizontal, the tangent plane of $z = f(x, y)$ at (a, b) is horizontal if and only if

Definition 6. $\left(\dfrac{\partial}{\partial x}\right)^2 f(x, y) = \dfrac{\partial^2}{\partial x^2} f(x, y) = f_{xx}(x, y)$ are all different notation for the same second partial derivative. Similarly, $\dfrac{\partial}{\partial x}\left(\dfrac{\partial}{\partial y} f(x, y)\right) = \dfrac{\partial^2}{\partial x \partial y} f(x, y) = f_{yx}(x, y)$ are all different notations for the same partial derivative.

Challenge 7. Let $f(x, y) = x^3 + x^2 y + y^2$. Compute $f_{xx}(x, y)$, $f_{xy}(x, y)$, $f_{yx}(x, y)$, and $f_{yy}(x, y)$. Notice anything? Hint: Compute $f_x(x, y)$ and $f_y(x, y)$ first.

Theorem 8. If f_{xy} and f_{yx} are continuous, then