Name:______________________

Notesheet. Section 7.4: Improper Integrals

Math 1220

Recall that

Theorem 1. For integrable $f(x) \geq 0$,

$$\int_a^b f(x)\ dx = \text{Area under the curve } y = f(x) \text{ from } a \text{ to } b.$$

Definition 2. We define <u>improper integrals</u> over unbounded intervals as follows

(a) $\int_a^\infty f(x)\ dx =$

(b) $\int_{-\infty}^b f(x)\ dx =$

Furthermore, we say an improper integral is

- <u>convergent</u> if
- <u>divergent</u> if

Challenge 3. (Review). Find the limits $\lim_{x\to\infty} f(x)$, $\lim_{x\to-\infty} f(x)$, and $\lim_{x\to 0} f(x)$. Draw a graph if necessary.

(a) $f(x) = e^{kx}$ for $k > 0$

(b) $f(x) = \ln|x|$

(c) $f(x) = \dfrac{2x^2 + x - 1}{3x^3 - 1}$

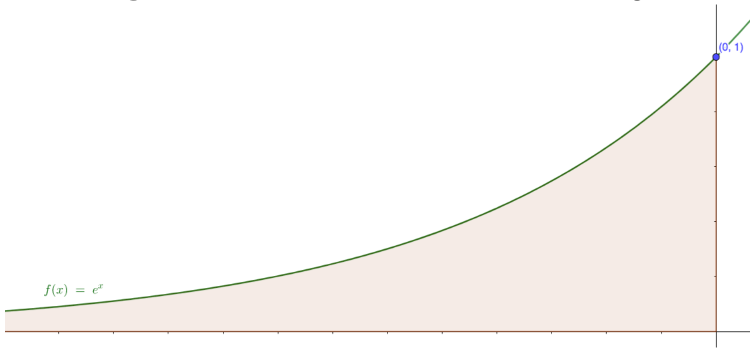
(d) $f(x) = \dfrac{2x^3 + x - 1}{3x^3 - 1}$

(e) $f(x) = \dfrac{2x^4 + x - 1}{3x^3 - 1}$

Challenge 4. Using the definition of an improper integral, evaluate $\int_1^\infty x^{-2}\ dx$ if it is convergent.

Challenge 5. Is $\int_{-\infty}^1 \frac{1}{x}\ dx$ convergent or divergent?

Challenge 6. Find the area of the shaded region where the curve is given by $f(x) = e^x$.



Theorem 7. Note that

$$= \int_{-\infty}^{c} f(x)\ dx + \int_{c}^{\infty} f(x)\ dx$$

for any real number c.