Name:________________________

Notesheet. Section 7.4: Improper Integrals Part II

Math 1220

Definition 1. If there is a real number c such that *both* of $\int_{-\infty}^{c} f(x)\ dx$ and $\int_{c}^{\infty} f(x)\ dx$ are convergent, then we say $\int_{-\infty}^{\infty} f(x)\ dx$ is

Furthermore, we compute it as

$$\int_{-\infty}^{\infty} f(x)\ dx =$$

Otherwise, if $\int_{-\infty}^{c} f(x)\ dx$ or $\int_{c}^{\infty} f(x)\ dx$ are divergent for some c, then we say $\int_{-\infty}^{\infty} f(x)\ dx$ is

Challenge 2. Evaluate $\int_{-\infty}^{\infty} \frac{2x}{x^2+1}\ dx$ if it converges. Also try $\int_{-\infty}^{\infty} \frac{2x}{(x^2+1)^2}\ dx$.

Challenge 3. Determine if $\int_{-\infty}^{\infty} x\ dx$ converges. Before using any calculus, what does your intuition tell you?

Challenge 4. True or False? $\displaystyle\int_{-\infty}^{\infty} f(x)\ dx = \lim_{t\to\infty} \int_{-t}^{t} f(x)\ dx$?

Challenge 5. Evaluate $\displaystyle\int_{-\infty}^{\infty} xe^{-\frac{1}{2}x^2}\ dx$ if it converges.

Challenge 6. Using some of what we have already seen, for which real numbers p is the following integral convergent?

$$\int_{1}^{\infty} \frac{1}{x^p}\ dx$$

Remark 7. *Later we will use the fact that*

$$\int_{-\infty}^{\infty} e^{-\frac{1}{2}x^2}\ dx =$$

for many probability problems.