

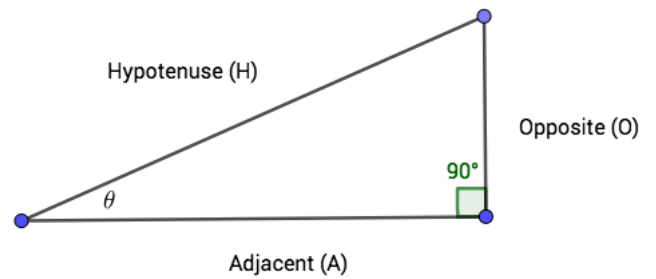
Name: \_\_\_\_\_

# Notesheet. Section 12.2: Trigonometric Functions

Math 1220

**Definition 1.** Given a right triangle with angle  $\theta$  marked below, we define our trigonometric functions as follows:

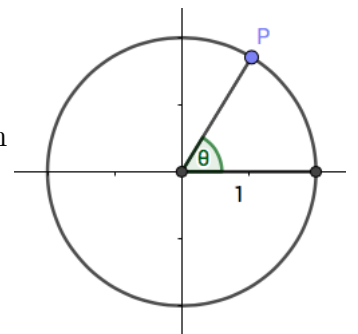
- $\sin \theta =$
- $\cos \theta =$
- $\tan \theta =$
- $\csc \theta =$
- $\sec \theta =$
- $\cot \theta =$



Useful mnemonic: SOHCAHTOA

**Challenge 2.** Consider a right triangle with  $O = 5$ ,  $A = 12$ . What is  $\sin \theta$  equal to? What is  $\cos \theta$  equal to?

**Theorem 3.** Let  $P = (x, y)$  be a point on the unit circle with angle  $\theta$  from the  $x$ -axis. Then,



**Challenge 4.** If  $\theta = \frac{\pi}{4}$  radians, then  $P = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ . If  $\theta = \frac{\pi}{6}$ , then  $P = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ .

(a) Using geometry, figure out the  $(x, y)$ -coordinates of  $P$  when  $\theta = \frac{\pi}{3}$ . Hint: Draw a picture!

(b) What are the  $(x, y)$ -coordinates of  $P$  when  $\theta = -\frac{7\pi}{6}$ ? What about when  $\theta = \frac{11\pi}{4}$ ? Hint:

$$-\frac{7\pi}{6} = -\pi - \frac{\pi}{6}.$$

**Theorem 5** (Useful properties of sine and cosine). (a) For any value of  $\theta$ ,  $\sin \theta$  and  $\cos \theta$  are bounded by the inequalities

(b)  $\sin(\theta + 2\pi) =$   $\cos(\theta + 2\pi) =$

(c) The graphs of  $\sin \theta$ ,  $\cos \theta$ , and  $\tan \theta$  are given by

(d)  $\sin(-\theta) =$   $, \sin \theta = 0 \iff$

(e)  $\cos(-\theta) =$   $, \cos \theta = 0 \iff$

(f)  $\tan(-\theta) =$   $, \tan \theta = 0 \iff$   $, \tan(\theta + \pi) =$

**Challenge 6.** Find all values of  $\theta$  such that  $\csc \theta = -2$ .