Name:_____

Notesheet. Section 11.5: Power Series and Taylor Series

Math 1220

Remark 1. *Recall that, for a differential function $f(x)$, we can* approximate *$f(x)$ near $x = a$ with the* linear equation

$$f(x) \approx f(a) + f'(a)(x - a)$$

We want to take this idea further.

Definition 2. *A power series centered at $x = a$ is a series of the form*

Challenge 3. Are the following series power series?

(a) $\displaystyle\sum_{n=0}^{\infty} x^n$

(b) $\displaystyle\sum_{n=0}^{\infty} x^{n-1}$

(c) $\displaystyle\sum_{n=0}^{\infty} x^2(x-1)^n$

Challenge 4. When is $\displaystyle\sum_{n=0}^{\infty} x^n$ convergent and when is it divergent?

Definition 5. *(a) The interval of convergence (IoC) for a power series*

(b) The radius of convergence (RoC) is defined to be

Theorem 6. The radius of convergence for $\sum_{n=0}^{\infty} a_n(x-a)^n$ is given by

$$R =$$

Challenge 7. Find the radius of convergence and the interval of convergence for the following power series:

(a) $\sum_{n=0}^{\infty} x^n$

(b) $\sum_{n=0}^{\infty} n!(x-1)^n$

(c) $\sum_{n=0}^{\infty} \frac{n^3(x+1)^n}{(n+1)!}$

(d) $\sum_{n=0}^{\infty} (2x+6)^n$