# Notesheet. Section 11.4 Part 2: Series with Positive Terms 

Math 1220

Challenge 1. Determine the convergence of the following series:
(a) $\sum_{n=1}^{\infty} \frac{5 n-1}{2 n^{3}+4 n+3}$
(b) $\sum_{n=1}^{\infty} \frac{9+\sin (n)}{n^{3}}$

Other series to try at home:

$$
\sum_{n=1}^{\infty} \frac{9+\sin (n)}{n}, \sum_{n=2}^{\infty} \frac{\pi^{n}}{e^{n}-1}, \sum_{n=2}^{\infty} \frac{1}{2 \sqrt{n^{2}-1}}, \sum_{n=1}^{\infty} \frac{1}{n^{n}}\left(\text { Hint: } n^{n} \geq n^{2}\right)
$$

Challenge 2. True or False?
(a) We can apply the comparison test on $\sum_{n=1}^{\infty} \frac{(-1)^{n}}{n^{2}}$
(b) If $\sum a_{n}, \sum b_{n}$, and $\sum c_{n}$ are all series with positive terms, $\sum a_{n}$ is convergent, and $b_{n}+c_{n} \leq a_{n}$ for all $n$, then $\sum b_{n}$ and $\sum c_{n}$ are convergent.
(c) If $\sum a_{n}, \sum b_{n}$, and $\sum c_{n}$ are all series with positive terms, $\sum a_{n}$ is divergent and $b_{n}+c_{n} \geq a_{n}$ for all $n$, then $\sum b_{n}$ and $\sum c_{n}$ are divergent.

Remark 3. A heuristic approach to solving series convergence problems in this class goes as follows

$$
\begin{aligned}
& \sum a_{n} \text { converges } ? \rightarrow\left\{\begin{array}{l}
\text { geo series } \sum_{n=0}^{\infty} a^{\infty} ? \xrightarrow{\text { Yes }} \xrightarrow{\text { feleseoping series? } \xrightarrow{\text { Yes }}} \begin{array}{l}
\text { Y-series } \sum_{n=1}^{\infty} \frac{1}{n^{p}} \xrightarrow{\text { Yes }}
\end{array}
\end{array}\right.
\end{aligned}
$$

