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# Notesheet. Section 11.3: Series 

Math 1220

Definition 1. A series is

Definition 2. Given a sequence $\left\{a_{n}\right\}_{n=k}^{\infty}$, the $N$ th partial sum for $N \geq k$ is

Definition 3. We say the series $\sum_{n=k}^{\infty} a_{n}$ is convergent if We say $\sum_{n=k}^{\infty} a_{n}$ is divergent if

Challenge 4. Is $\sum_{n=2}^{\infty} \frac{3}{2^{n}}$ convergent or divergent?

Definition 5. The series above is a geometric series. A geometric series with ratio $r$ is a series of the form

Theorem 6. A geometric series $\sum_{n=0}^{\infty} a r^{n}$ is convergent with sum and it is divergent if

Challenge 7. Express the decimal $0.131313 \cdots$ as a fraction of integers. (Hint: Write $0.1313 \cdots$ as a geometric series.)

Theorem 8. If $\sum_{n=1}^{\infty} a_{n}$ and $\sum_{n=1}^{\infty} b_{n}$ are convergent infinite series and $c$ is a constant, then
(a) $\sum_{n=1}^{\infty} c a_{n}=$
(b) $\sum_{n=1}^{\infty}\left(a_{n} \pm b_{n}\right)=$

Definition 9. A $\underline{\text { telescoping series }}$ is a series $\sum_{n=k}^{\infty} a_{n}$ such that

Challenge 10. Determine the convergence of the following (telescoping) series using the partial sum definition.
(a) $\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{n+1}\right)$
(b) $\sum_{n=1}^{\infty} \frac{1}{n^{2}+n}$

