Notesheet. Section 11.3: Series

Math 1220

Definition 1. A <u>series</u> is

Definition 2. Given a sequence $\{a_n\}_{n=k}^{\infty}$, the <u>Nth partial sum</u> for $N \ge k$ is

Definition 3. We say the series $\sum_{n=k}^{\infty} a_n$ is <u>convergent</u> if We say $\sum_{n=k}^{\infty} a_n$ is divergent if

Challenge 4. Is $\sum_{n=2}^{\infty} \frac{3}{2^n}$ convergent or divergent?

Definition 5. The series above is a geometric series. A geometric series with ratio r is a series of the form

Theorem 6. A geometric series $\sum_{n=0}^{\infty} ar^n$ is convergent with sum

and it is divergent if

Challenge 7. Express the decimal $0.131313\cdots$ as a fraction of integers. (Hint: Write $0.1313\cdots$ as a geometric series.)

Theorem 8. If $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are convergent infinite series and c is a constant, then (a) $\sum_{n=1}^{\infty} ca_n =$ (b) $\sum_{n=1}^{\infty} (a_n \pm b_n) =$

Definition 9. A <u>telescoping series</u> is a series $\sum_{n=k}^{\infty} a_n$ such that

Challenge 10. Determine the convergence of the following (telescoping) series using the partial sum definition.

(a)
$$\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1} \right)$$

(b)
$$\sum_{n=1}^{\infty} \frac{1}{n^2 + n}$$