Name:________________________

Notesheet. Section 11.3+11.4: Series and Series with Positive Terms

Math 1220

Challenge 1. Find the sum $\displaystyle\sum_{n=1}^{\infty}\left(\frac{2}{n^2+n}+\frac{3}{2^n}\right)$.

Definition 2. The Harmonic series is the series

Theorem 3. (a) If $\displaystyle\sum_{n=1}^{\infty} a_n$ converges, then $\displaystyle\lim_{n\to\infty} a_n =$

(b) (Test for Divergence) If $\displaystyle\lim_{n\to\infty} a_n$ does not exist or $\displaystyle\lim_{n\to\infty} a_n \neq 0$, then $\displaystyle\sum_{n=1}^{\infty} a_n$

Challenge 4. Determine if $\displaystyle\sum_{n=1}^{\infty} e^{\sin(n\pi)}$ converges.

Other series to know and try at home: $\sum_{n=1}^{\infty}(-1)^n, \sum_{n=1}^{\infty}(-1)^n \frac{n^2}{n^2+1}, \sum_{n=1}^{\infty} \frac{n}{\sqrt{2n^2+10}}, \sum_{n=1}^{\infty} \tan(n)$.

Theorem 5. (Integral Test). Suppose f is a continuous, positive, and decreasing function on $[k, \infty)$. If $f(n) = a_n$ for all integers $n \geq k$, then

Challenge 6. Use the integral test to show that the Harmonic series is divergent. In general, for p a constant, use the integral testto deduce the convergence of

$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$

Theorem 7. (The Comparison Test) Suppose $\sum_{n=k}^{\infty} a_n$ and $\sum_{n=k}^{\infty} b_n$ are series with positive terms.

(a) If $\sum_{n=k}^{\infty} b_n$ is convergent and $a_n \leq b_n$ for all n, then $\sum_{n=k}^{\infty} a_n$

(b) If $\sum_{n=k}^{\infty} b_n$ is divergent and $a_n \geq b_n$ for all n, then $\sum_{n=k}^{\infty} a_n$

Challenge 8. Determine the convergence of

(a) $\sum_{n=1}^{\infty} \frac{1}{n^{1/3} - 0.1}$

(b) $\sum_{n=0}^{\infty} \frac{2^n}{3^n + 2}$