Name:________________________

Notesheet. Section 11.2: Infinite Sequences

Math 1220

Definition 1. An infinite sequence $\{a_n\}$ is a function whose domain is

The terms of the sequence are

So, the nth term is

Remark 2. *Sometimes* $\{a_n\}$ *is denoted* $\{a_n\}_{n=1}^{\infty}$. *A sequence also can begin at any natural number* k*, e.g.*

(a) $\left\{\dfrac{n}{n+1}\right\}_{n=1}^{\infty} =$

(b) $\{(-1)^n n\}_{n=2}^{\infty} =$

(c) $\{(2n+1)!\}_{n=0}^{\infty} =$

Challenge 3. Find a formula for the nth term of

(a) $\{a_n\}_{n=1}^{\infty} = \left\{\dfrac{1}{2}, \dfrac{1}{4}, \dfrac{1}{6}, \dfrac{1}{8}, \dfrac{1}{10}, \ldots\right\}$

(b) $\{a_n\}_{n=1}^{\infty} = \left\{-\dfrac{4}{5}, \dfrac{8}{8}, -\dfrac{16}{11}, \dfrac{32}{14}, -\dfrac{64}{17}, \ldots\right\}$

Definition 4. The sequence $\{a_n\}_{n=k}^{\infty}$ is called convergent if

The sequence is called divergent if

Theorem 5. The "limit laws" hold for sequences as well: Assume

$$\lim_{n\to\infty} a_n = A < \infty \text{ and } \lim_{n\to\infty} b_n = B < \infty$$

(a) For c a constant, $\lim_{n\to\infty} ca_n =$

(b) $\lim_{n\to\infty} (a_n \pm b_n) =$

(c) $\lim_{n\to\infty} a_n b_n =$

(d) $\lim_{n\to\infty} \frac{a_n}{b_n} =$

Challenge 6. Determine if the following sequences converge or diverge. If the converge, give the limit.

(a) $\{a_n\} = \left\{\left(\frac{1}{2}\right)^n\right\}$

(b) $\{b_n\} = \left\{\frac{2n^2+n}{3n^2+1}\right\}$

(c) $\{c_n\} = \left\{\frac{2n^2+n}{3n^3+1}\right\}$

(d) $\{d_n\} = \left\{\frac{2n^2+n}{3\sqrt{n}+1}\right\}$

Theorem 7. The rate of growth of $n! \gg e^n \gg n^k$. In other words

- $\lim_{n\to\infty} \frac{e^n}{n!} = 0 = \lim_{n\to\infty} \frac{n^k}{n!} = \lim_{n\to\infty} \frac{n^k}{e^n}$
- $\lim_{n\to\infty} \frac{n!}{e^n}$, $\lim_{n\to\infty} \frac{n!}{n^k}$, and $\lim_{n\to\infty} \frac{e^n}{n^k}$ all do not exist and the corresponding sequences tend towards infinity.

Challenge 8. Of the microprocessors manufactured by a microelectronics firm for use in regularing fuel consumption in automobiles, 1.5% are defective. It can be shown that the probability of getting at least one defective microprocessor in a random sample of n microprocessors is $f(n) = 1 - (0.985)^n$. Consider the sequence $\{a_n\}$ defined by $a_n = f(n)$. What is $\lim_{n\to\infty} a_n$ and interpret the result.