Name:________________________

Notesheet. Section 10.1: Probability Distributions and Random Variables

Math 1220

Remark 1. *Recall that the probability of an event E occurring (such as getting a ⚁ on a 6-sided die) is given by*

$$P(E) =$$

What is the probability of rolling an odd number on a 6-sided die?

Definition 2. (a) The sample space S of an experiement is

(b) An event is

(c) A random variable (RV) X is a

and it is called continuous if
and discrete if

(d) A probability density function (PDF) of a random variable X in an interval I is a function $f(x)$ such that

(i)

(ii)

Challenge 3. Are the following functions valid PDFs?

(a) For rolling a 6-sided die, let $f(t) = \dfrac{1}{6}$ for $t = 1, \dots, 6$.

(b) For rolling a 6-sided die, let $f(6) = 1$ and $f(t) = 0$ for all other t.

(c) $S = [0, 1]$. Let $f(x) = x$.

(d) $S = [0, 1]$. Let $f(x) = \dfrac{e^x}{e - 1}$.

Challenge 4. Find the value of k such that $f(x) = ke^{-x}$ over $S = [0, \infty)$ is a PDF.

Definition 5. Let $[a, b] \subset I$, the interval for a continuous random variable X. Then, the event $E =$ "$a \leq X \leq b$" has probability

$$P(a \leq X \leq b) =$$

Challenge 6. $f(x) = \dfrac{1}{9}x^2$ on $[0, 3]$ is a PDF. Evaluate the following probabilities

(a) $P(1 \leq X \leq 3)$

(b) $P(1 \leq X)$

(c) $P(X = 1)$

(d) $P(x < 1)$

Challenge 7. The life expectancy (in years) of a TV is a continuous RV with PDF

$$f(t) = \frac{1}{2}e^{-\frac{t}{2}}, \quad (0 \leq t < \infty)$$

Find the probability that a randomly chosen TV will last more than 2 years. (This distribution is called an exponential density function.)