Name:______________________

Notesheet. Section 10.1+10.2: Joint Distributions, Expected Value, and Standard Deviation

Math 1220

Definition 1. A joint probability dnesity function of the random variables X and Y on a region D is a function $f(x, y)$ such that

Theorem 2. The event $E =$ "(X, Y)" in R has probability

$$P[(X, Y) \text{ in } R] =$$

Challenge 3. Show that

$$f(x, y) = \frac{1}{4}(x + 2y); D = \{0 \leq x \leq 2; 0 \leq y \leq 1\}$$

is a joint probability density function. Find the probability $P(0 \leq X \leq 1; 0.5 \leq Y \leq 1)$.

Definition 4. The expected value or mean $E(X)$ of a continuous random variable X on an interval I is given by

$$E(X) =$$

and refers to "the value one would expect to find if one repeated the experiment infinitely many times."

For a discrete RV Y with possible values $\{y_1, \ldots, y_n\}$ and corresponding probabilities $\{p_1, \ldots, p_n\}$,

$$E(X) =$$

Challenge 5. Let $f(x) = \dfrac{e^x}{e-1}$ be the PDF for X on $I = [0, 1]$. Find $E(X)$.

Definition 6. The variance of a random variable X on I is given by

$$\text{Var}(X) =$$

and the standard deviation of X is

$$\sigma =$$

Remark 7. *If we rewrite $E(X) = \mu$, since $E(X)$ is a constant, then we get an easier formula for variance given by*

$$\text{Var}(X) =$$

Challenge 8. Let $f(x) = \dfrac{e^x}{e-1}$ be the PDF for RV X on $I = [0, 1]$. Find $\text{Var}(X)$.

Challenge 9. Assume X is a *RV* on I with $\text{Var}(X) = 2$ and $\int_I x^2 f(x)\, dx = 11$. Find $E(X)$.