Name:________________________

Notesheet. Section 6.5: Evaluating Definite Integrals

Math 1210

Theorem 1 (Properties of Definite Integrals)**.** Let f and g be integrable functions. Then,

(a) $\int_a^a f(x)\,dx =$

(b) $\int_a^b f(x)\,dx =$

(c) $\int_a^b kf(x)\,dx =$

(d) $\int_a^b [f(x) \pm g(x)]\,dx =$

(e) $\int_a^b f(x)\,dx + \int_b^c f(x)\,dx =$

Challenge 2. Evaluate

$$\int_0^4 \sqrt{2x+1}\,dx$$

by first finding $\int \sqrt{2x+1}\,dx$ and then applying the Fundamental Theorem of Calculus.

Theorem 3 (Substitution for Definite Integrals)**.** If g' is continuous on $[a, b]$ and f is continuous on the range of $g(x)$, then

$$\int_a^b f(g(x))g'(x)\,dx = \int$$

Challenge 4. Find the area under the graph of $f(x) = 8xe^{x^2}$ on the interval $[0, 2]$.

Challenge 5. If f is continuous and $\int_0^4 f(x)\,dx = 10$, find $\int_0^2 f(2x)\,dx$. If g is continuous and $\int_0^9 g(x)\,dx = 4$, find $\int_0^3 xg(x^2)\,dx$.

Theorem 6. Given a function f that is integrable on $[a, b]$, the average value of f over $[a, b]$ is

Challenge 7. The height of the water in Swan Lake is measured periodically over the course of a year. The height $h(t)$ in inches as a function of time t in weeks is modelled by

$$h(t) = 40 + \frac{1}{(t+1)^2}.$$

What is the average height of the lake water that year?