Name:________________________

Notesheet. Section 6.3: Area and the Definite Integral

Math 1210

Challenge 1. (Together) An oil company produces a constant rate of $F'(t) = 1.2$ million barrels per year. How many barrels does it produce in 4 years? (That's $F(4)$). How many barrels does it produce in t years? (That's $F(t)$).

Challenge 2. (Together) What if the rate of oil production $F'(t)$ is not constant? How can we approximate the amount of oil produced in t years? (That's $F(t)$ again).

Theorem 3 (Area under Graph of a Function)**.** If f is a nonnegative continuous function on $[a, b]$, then the area A of the region under the graph is

$$A = \lim_{n\to\infty}$$

where $x_1, , , x_n$ are points from the n subintervals of $[a, b]$ of equal width $\Delta x = \dfrac{b-a}{n}$.

Definition 4. If f is a function defined on $[a, b]$, and

exists for all choices of points $x_1, , , x_n$ in the subintervals, then this limit is the area under the curve and it is called the definite integral and it is denoted $\int_a^b f(x)\, dx$.

Theorem 5. If f is defined on $[a, b]$ and continuous, then $\int_a^b f(x)\, dx$ exists. (We say "f is integrable on $[a, b]$.")

Challenge 6. What does $\int_2^6 (x^2+1)\, dx$ mean in terms of area? Draw a picture. Approximate the area $\int_2^6 (x^2+1)\, dx$ by cutting $[2, 6]$ into 4 equal intervals. Now compute $\int_2^6 (x^2+1)\, dx$ a different way. Was the approximation accurate?

Challenge 7. What happens if the function dips down below the x-axis? What is the area under the curve $y = 4 - x$ on the interval $[0, 5]$? What is $\int_0^5 (4 - x)\, dx$?