

## Notesheet. Section 6.3: Area and the Definite Integral

## Math 1210

**Challenge 1.** An oil company produces a constant rate of 1.2 million barrels per year. How many barrels does it produce in 4 years? How many barrels does it produce in  $t$  years?

**Theorem 2** (Area under Graph of a Function). If  $f$  is a nonnegative continuous function on  $[a, b]$ , then the area  $A$  of the region under the graph is

$$A = \lim_{n \rightarrow \infty}$$

where  $x_1, \dots, x_n$  are points from the  $n$  subintervals of  $[a, b]$  of equal width  $\Delta x = \frac{b-a}{n}$ .

**Definition 3.** If  $f$  is a function defined on  $[a, b]$ , and

exists for all choices of points  $x_1, \dots, x_n$  in the subintervals, then this limit is the area under the curve and it is called the definite integral and it is denoted  $\int_a^b f(x) dx$ .

**Theorem 4.** If  $f$  is continuous on  $[a, b]$ , then  $\int_a^b f(x) dx$  exists. (We say “ $f$  is integrable on  $[a, b]$ .”)

**Challenge 5.** What does  $\int_2^6 (x^2 + 1) dx$  mean in terms of area? Draw a picture. Approximate the area  $\int_2^6 (x^2 + 1) dx$  by cutting  $[2, 6]$  into 4 equal intervals. Is this approximation accurate?

**Challenge 6.** What happens if the function dips down below the  $x$ -axis? What is the area under the curve  $y = 4 - x$  on the interval  $[0, 5]$ ? Using the definition, what is  $\int_0^5 (4 - x) dx$ ?

**Challenge 7.** Compute  $F(x) = \int (4 - x) dx$ . What is  $F(5) - F(0)$ ?