Notesheet. Section 5.2: Logarithms

Math 1210

Definition 1. We define $y = \log_b x$ to be the number such that

Challenge 2. What are the following logarithms? Try to justify your answer from the definition of $\log_b x$ above.

   (a) $\log_b m + \log_b n$

   (b) $\log_b m - \log_b n$

   (c) $n \log_b m$

   (d) $\log_b 1$

   (e) $\log_b b$

Challenge 3. Remembering theorem 5 on notesheet 5.1, can you figure out the following properties of $f(x) = \log_b x$?

- Its domain is

- Its range is

- Its graph always passes through the point

- It is continuous on

- If $b > 1$, then it is increasing on   . If $b < 1$, it is decreasing on   .

Challenge 4. What is $\log_b(b^x)$? What is $b^{\log_b x}$?

Challenge 5. $f(x) = \log_e x = \ln x$ is called the <u>natural logarithm</u> and is the inverse function to $g(x) = e^x$. In other words, for all x in the appropriate domain $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$. Can you graph the equation $f(x) = \ln(x)$?

Challenge 6. Write each of the following expressions as the logarithm of a single quantity

(a) $2 \ln a + 3 \ln b$

(b) $\ln 2 + \dfrac{1}{2} \ln(x+1) - 2 \ln(1 + \sqrt{x})$

Challenge 7. Expand and simplify the following expressions.

(a) $\log_{10} x(x+1)^4$

(b) $\ln x e^{-x^2}$

(c) $\ln \dfrac{x^2}{\sqrt{x}(1+x)^2}$

Challenge 8. Use logarithms to solve the equation $e^{0.4t} = 8$ for t.