Name:________________________

Notesheet. Section 4.4+4.5: Optimization

Math 1210

Definition 1. Let f be a function. Then, we say a value $f(c)$ is an absolute maximum value (or global maximum value) of f if

Theorem 2. If f is continuous on a closed interval $[a, b]$, then f acheives an absolute maximum value and an absolute minimum value on $[a, b]$.

Challenge 3. Find the absolute maximum and minimum values of the function

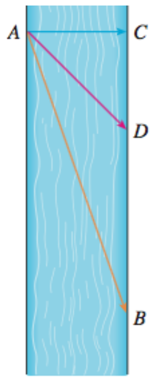
$$f(x) = x^3 - 3x^2 + 1, \qquad -\frac{1}{2} \leq x \leq 4$$

Challenge 4. A manufacturer of tennis rackets find that the total cost $C(x)$ (in dollars) of manufacturing x rackets/day is given by $C(x) = 400 + 4x + 0.0001x^2$. Each racket can be sold at a price of p dollars, where p is related to x by the demand equation $p = 10 - 0.0004x$. If all rackets that are manufactured can be sold, find the daily level of production that will yield a maximum profit for the manufacturer. (Hint: $\sqrt{15} \approx 3.9$).

Challenge 5. Find the global minimum and maximum of $f(x) = |x - 3|$ on $[1, 4]$.

Challenge 6. Find the point on the parabola $y = x^2 + 1$ that is closest to the point $(0, 2)$.

Challenge 7. A woman launches her boat from point A on a bank of a straight river, 1 km wide, and wants to reach point B, 10km downstream on the opposite bank, as quickly as possible. She could row her boat directly across the river to point C and then run to B, or she could row directly to B, or she could row to some point D between C and B, and then run to B. If she can row 6 km/h and run 10 km/h, where should she land to reach B as soon as possible? (We assume that the speed of the water is negligible compared with the speed at which the woman rows.) Hint: $\frac{5}{24} \approx 0.20833$, $\frac{37}{40} = 0.925$, and $\sqrt{101} \approx 10.05$.



Challenge 8. A grain silo has the shape of a right circular cylinder surmounted by a hemisphere (and the silo has no base). If the silo is to have a capacity of 18π m^3, find the radius and height of the silo that requires the least amount of material to construct.