Name:________________________

Notesheet. Section 4.2: Applications of the Second Derivative

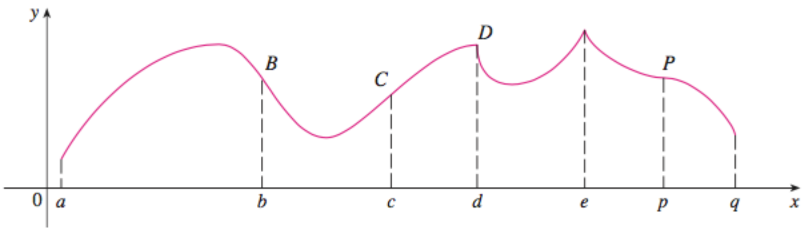
Math 1210

Definition 1. Let a function f be differentiable on an interval (a, b). Then,

(a) f is <u>concave upward</u> on (a, b) if

(b) f is <u>concave downward</u> on (a, b) if

Challenge 2. Where is the following graph concave upwards and where is it concave downwards?



Challenge 3. Consider the function $f(x) = x^3$. Where is f concave upward and where is it concave downward? What can we say about $f''(x)$ on these intervals?

Theorem 4. Let f be twice differentiable on an interval (a, b). Then,

(a) If $f''(x) > 0$ for each value of x in (a, b), then

(b) If $f''(x) < 0$ for each value of x in (a, b), then

Definition 5. A <u>point of inflection</u> $(a, f(a))$ on a graph of a function f is

Challenge 6. What is the second derivative of $f(x) = x^3$ at the point(s) of inflection? What is the second derivative at the point(s) of inflection of

$$g(x) = \begin{cases} \dfrac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

Finally, does $h(x) = x^4$ have any inflection points?

Theorem 7. If $(a, f(a))$ is an inflection point for the graph of f, then $f''(a) = 0$ **or** $f''(a)$ **does not exist.**

Challenge 8. Consider the function $f(x) = x^4 - 4x^3$. Where is f concave up and concave down? Where are its points of inflection? Find the points where $f'(x) = 0$ and evaluate the second derivative of f at these points.

Theorem 9. Let f be a twice differentiable function. Then, if $f'(c) = 0$ and

(a) $f''(c) < 0$, then

(b) $f''(c) > 0$, then

(c) $f''(c) = 0$, then