Name:________________________

Notesheet. Section 3.6: Implicit Differentiation

Math 1210

Definition 1. An explicit relationship between independent variable x and a dependent variable y is a relationship of the form $y = f(x)$, such as $y = \sqrt{x^3 + 1}$ or $y = 5x^2 + 7x + 1$. An implicit relationship between an independent variable x and dependent variable y is

Challenge 2. What is an equation whose solution is a circle of radius 2 centered at the origin, $(0, 0)$. Is this equation an implicit or explicit relationship between x and y?

Challenge 3. If $x^2 + y^2 = 25$, what is $\frac{dy}{dx}$? What is an equation of the tangent line to the circle $x^2 + y^2 = 25$ at the point $(3, 4)$? Hint: Do not solve for y in terms of x. Instead, assume y is a function of x, say $y = f(x)$, and use the chain rule. Your final answer may be in terms of x and y.

Definition 4. The process of finding the derivative $\frac{dy}{dx}$ from an implicit relationship between x and y is called implicit differentiation.

Challenge 5. Let $x^3 + y^3 = 6xy$ be the "folium of Descartes". What is y'? Find the tangent line to the folium at the point $(3, 3)$. At what points in the first quadrant is the tangent line horizontal?

Challenge 6. In the "Lots of Derivatives" notesheet, you were asked the following question. Air is being pumped into a spherical weather balloon. At any time t, the volume of the balloon is $V(t)$ and its radius is $r(t)$. Recall that, for a sphere, $V = \frac{4}{3}\pi r^3$. What is $\frac{dV}{dt}$ in terms of $r(t)$ and $r'(t)$?

Definition 7. In the above problem, finding $\frac{dV}{dt}$ required using $r'(t)$ in your final answer. This is a related rate. A <u>related rate</u> is

Challenge 8. A water tank has the shape of an inverted circular cone with base radius 2m and height 4m. If water is being pumped into the tank at a rate of 2 cubic meters/minute, find the rate at which the water level is rising when the water is 3m deep.