

Name: _____

Notesheet. Sections 3.3 (More Derivatives)

Math 1210

Theorem 1. Recall the chain rule, which says

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

or, alternatively

$$\frac{d}{dx}[(f \circ g)(x)] = f'(g(x)) \cdot g'(x).$$

Challenge 2. Using the chain rule, compute the derivative of $f(x) = (1 - x)^2$. Make sure it agrees with the answer when you expand $(1 - x)^2$. Also make sure it agrees with the product rule when thinking of $f(x) = g(x)h(x)$ with $g(x) = (1 - x)$ and $h(x) = (1 - x)$.

Challenge 3. If $F(x) = f(g(x))$ where $f(-2) = 8$, $f'(-2) = 4$, $f'(5) = 3$, $g(5) = -2$, and $g'(5) = 6$, find $F'(5)$.

Challenge 4. What is the derivative of $f(x) = (2x - 5)^4(8x^2 - 5)^{-2}$?

Challenge 5. Find an equation for the tangent line to the curve $y = \frac{|x|}{\sqrt{2-x^2}}$ at $(1, 1)$.

Challenge 6. Find the points on the graph of $f(x) = \frac{1}{3}(x^2 - 9)(x - 3)$ where the tangent line is horizontal.

Challenge 7. A manufacturer produces bolts of fabric with a fixed width. The quantity q of this fabric (in yards) that is sold is a function of the selling price p (in dollars per yard), so we can write $q = f(p)$. Then, the total revenue earned with selling price p is $R(p) = pf(p)$.

- (a) What does it mean to say that $f(20) = 10,000$ and $f'(20) = -350$?
- (b) Assuming the values in part (a), find $R'(20)$ and interpret your answer.