Name:_____

Notesheet. Section 2.5 (Continuity) Part II

Math 1210

Definition 1. What is a polynomial function and a rational function?

Theorem 2.

Every polynomial function is continuous at every point x.

Every rational function $f(x) = \dfrac{p(x)}{q(x)}$ is continuous at every point x provided that $q(x) \neq 0$.

Proof. Challenge!

Challenge 3. For which values of x is the function $g(x) = \dfrac{29x^{17} + 2x}{x^4 + 1}$ continuous? Use interval notation.

Challenge 4. In 2012, the postage rates for a package weighing x ounces were given by the

function

$$f(x) = \begin{cases} \$1.95 & 0 < x < 4 \\ \$2.12 & 4 \leq x < 5 \\ \$2.29 & 5 \leq x < 6 \\ \vdots & \\ \$3.48 & 12 \leq x < 13 \\ \$3.65 & x = 13 \end{cases}$$

Where is $f(x)$ discontinuous? Throwback: what is $\lim_{x \to 5^+} f(x)$? What is $\lim_{x \to 5^-} f(x)$?

Theorem 5 (Intermediate Value Theorem)**.** If f is a continuous function on a closed interval $[a, b]$ and M is any number between $f(a)$ and $f(b)$, then...

Challenge 6. Let f be a continuous function on the closed interval $[-1, 10]$ and suppose that $f(-1) > 0$ and $f(10) < 0$. Prove that there exists at least one solution to $f(x) = 0$.

Challenge 7. The oxygen content t days after organic waste has been dumped into a pond is given by

$$f(t) = 100 \left(\frac{t^2 + 10t + 100}{t^2 + 20t + 100} \right) \text{ percent of its normal level}$$

Show that $f(0) = 100$ and $f(10) = 75$. Must the pond have been at a level of 80% at some time? If so, at what time(s) was the oxygen content at 80%? Finally, what is $\lim_{t \to \infty} f(t)$ and what does it represent? (If necessary, $\sqrt{5} \approx 2.236$.)