

Name: _____

Notesheet. Section 2.4 Part II

Math 1210

Challenge 1. Consider the function $f(x) = 1 + \frac{1}{x^2}$. What is $f(10)$? What is $f(100)$? $f(10000)$? Is there a positive number N such that $f(N) \leq 1$?

Definition 2. We define the limit of $f(x)$ at infinity to be

Challenge 3. What is $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x^2}\right)$? What is $\lim_{x \rightarrow \infty} x$? What is $\lim_{x \rightarrow \infty} \frac{x+1}{4x}$? Harder question, can you figure out $\lim_{x \rightarrow \infty} \frac{x^2+1}{5x^2+3x-1}$? (Note that “does not exist (DNE)” is a valid answer.)

Challenge 4. Let $\lim_{x \rightarrow a} f(x) = L$ and $\lim_{x \rightarrow a} g(x) = M \neq 0$ for some a (including $\infty, -\infty$). Let c be some number. Keeping the examples above in mind, what are the following limits equal to in terms of L, M , and c ? $\lim_{x \rightarrow a} (c \cdot f(x))$, $\lim_{x \rightarrow a} (f(x) + g(x))$, $\lim_{x \rightarrow a} (f(x) \cdot g(x))$, and $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$. Given a number $b > 0$, what is $\lim_{x \rightarrow a} (f(x))^b$ assuming L^b is defined?

Challenge 5. The average cost per book in dollars incurred by TJ Publishing in printing x books is given by the average cost function

$$\bar{C}(x) = 4.5 + \frac{3000}{x}$$

Evaluate $\lim_{x \rightarrow \infty} \bar{C}(x)$ and interpret the meaning of this limit.

Definition 6. What is $\lim_{x \rightarrow 0} \frac{x}{x}$? What is $\lim_{x \rightarrow 0} \frac{x}{x^2}$? An indeterminate form is

Challenge 7. Evaluate $\lim_{x \rightarrow \infty} \frac{x}{1-x}$, $\lim_{x \rightarrow 5} \frac{x^2 - 4x - 5}{x - 5}$, and $\lim_{h \rightarrow 0} \frac{(h+1)^2 - 1}{h}$.